STA303: Artificial Intelligence

Deep Reinforcement Learning

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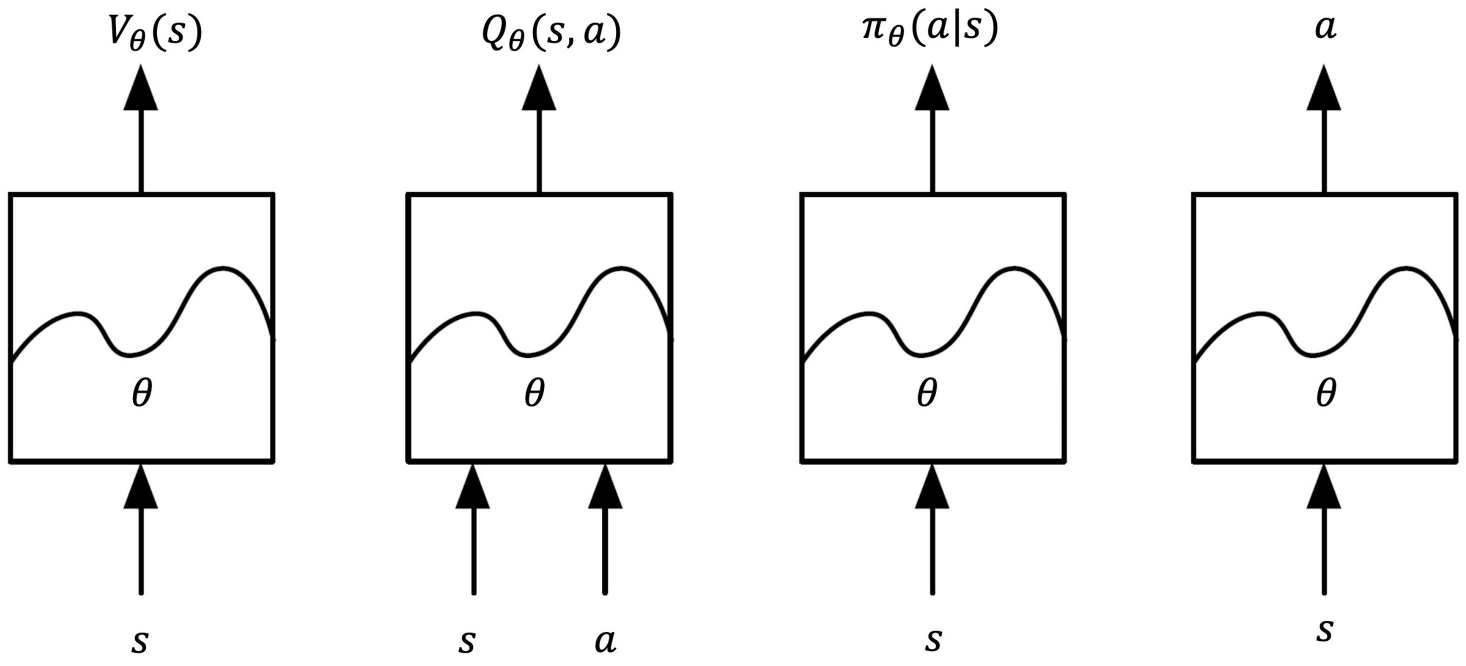
Part of slide credits: Weinan Zhang

Outline

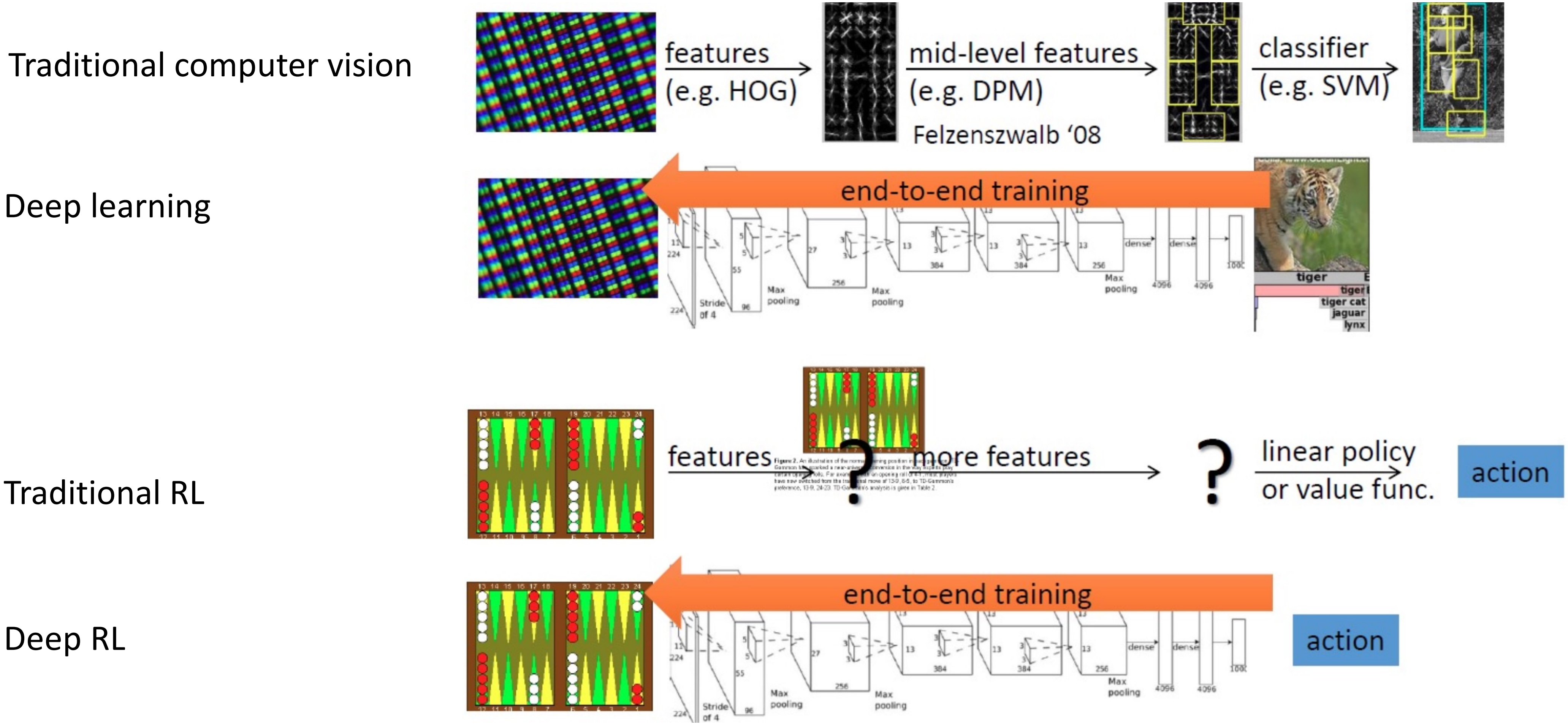
- Deep RL – Value methods
- Deep RL – Policy methods

Function approximation for value and policy



- What if we use deep neural networks directly to approximate these functions?

End-to-end reinforcement learning



- Deep RL enables RL algorithms to solve complex tasks in an end-to-end manner.

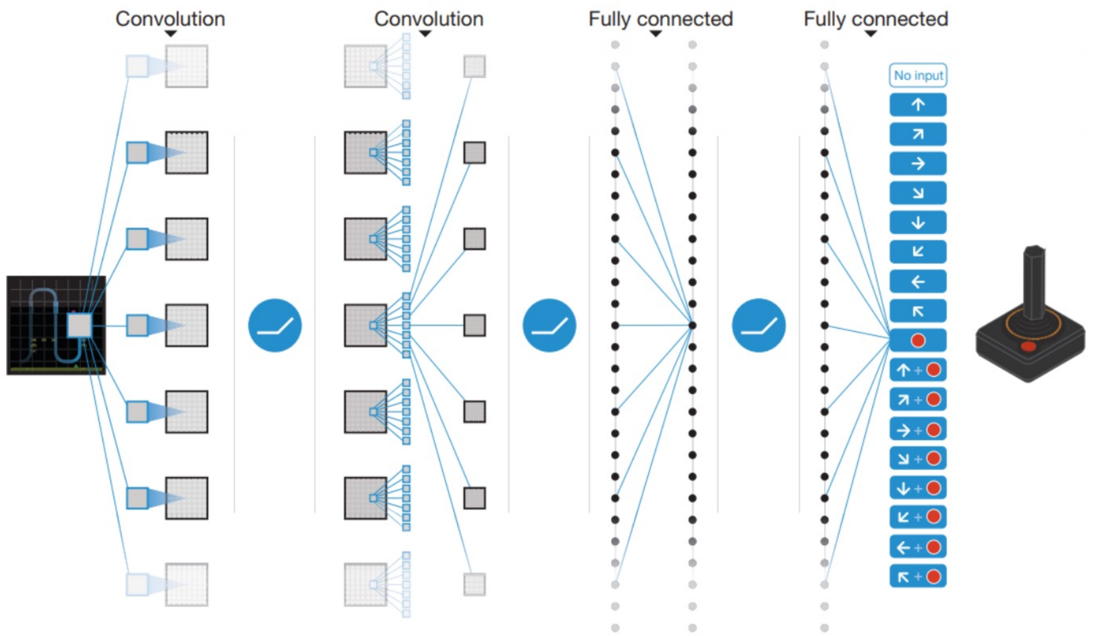
Slide from Sergey Levine. http://rail.eecs.berkeley.edu/deeprlcourse/static/slides/lec-1.pdf

Deep RL

- New challenges when we combine deep learning with RL?
  - Value functions and policies become deep neural networks
  - High-dimensional parameter space
  - Difficult to train stably
  - Prone to overfitting
  - Requires large amounts of data
  - High computational cost
  - CPU-GPU workload balance

Value methods: DQN

- Deep Q-Network (DQN)
  - Uses a deep neural network to approximate Q(s,a)
    - → Replaces the Q-table with a parameterized function for scalability
  - The network takes state s as input, outputs Q-values for all actions a simultaneously



Volodymyr Mnih, Koray Kavukcuoglu, David Silver et al. Playing Atari with Deep Reinforcement Learning. NIPS 2013 workshop.

DQN (cont.)

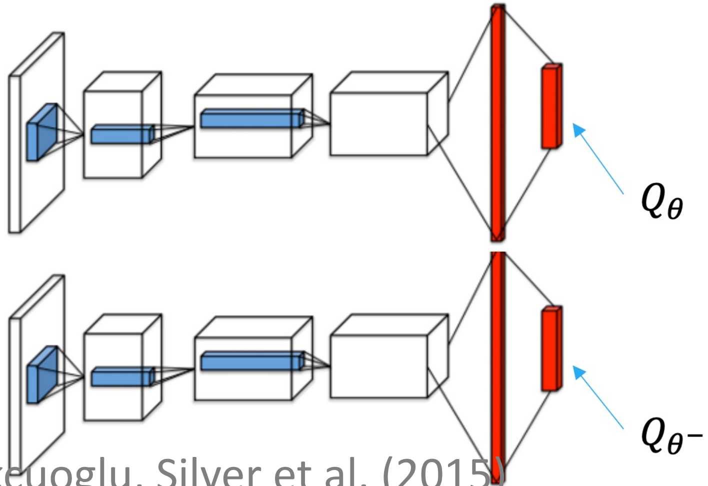
- Intuition: Use a deep neural network to approximate Q(s,a)
  - Instability arises in the learning process
    - Samples $\{(s_t, a_t, s_{t+1}, r_t)\}$ are collected sequentially and do not satisfy the i.i.d. assumption
    - Frequent updates of Q(s,a) cause instability
- Solutions: Experience replay
  - Store transitions $e_t = (s_t, a_t, s_{t+1}, r_t)$ in a replay buffer D
    Sample uniformly from D to reduce sample correlation
  - Dual network architecture: Use an evaluation network and a target network for improved stability

"Human-Level Control Through Deep Reinforcement Learning", Mnih, Kavukcuoglu, Silver et al. (2015)

Target network

- Target network $Q_{\theta^-}(s,a)$
  - Maintains a copy of the Q-network with older parameters θ^-
  - Parameters θ^- are updated periodically (every C steps) to match the evaluation network
- Loss Function (at iteration i)

$$L_i(\theta_i) = \mathbb{E}_{s_t,a_t,s_{t+1},r_t,p_t \sim D}\left[\frac{1}{2}\omega_t\left(r_t + \gamma \max_{a'} Q_{\theta_i^-}(s_{t+1},a') - Q_{\theta_i}(s_t,a_t)\right)^2\right]$$



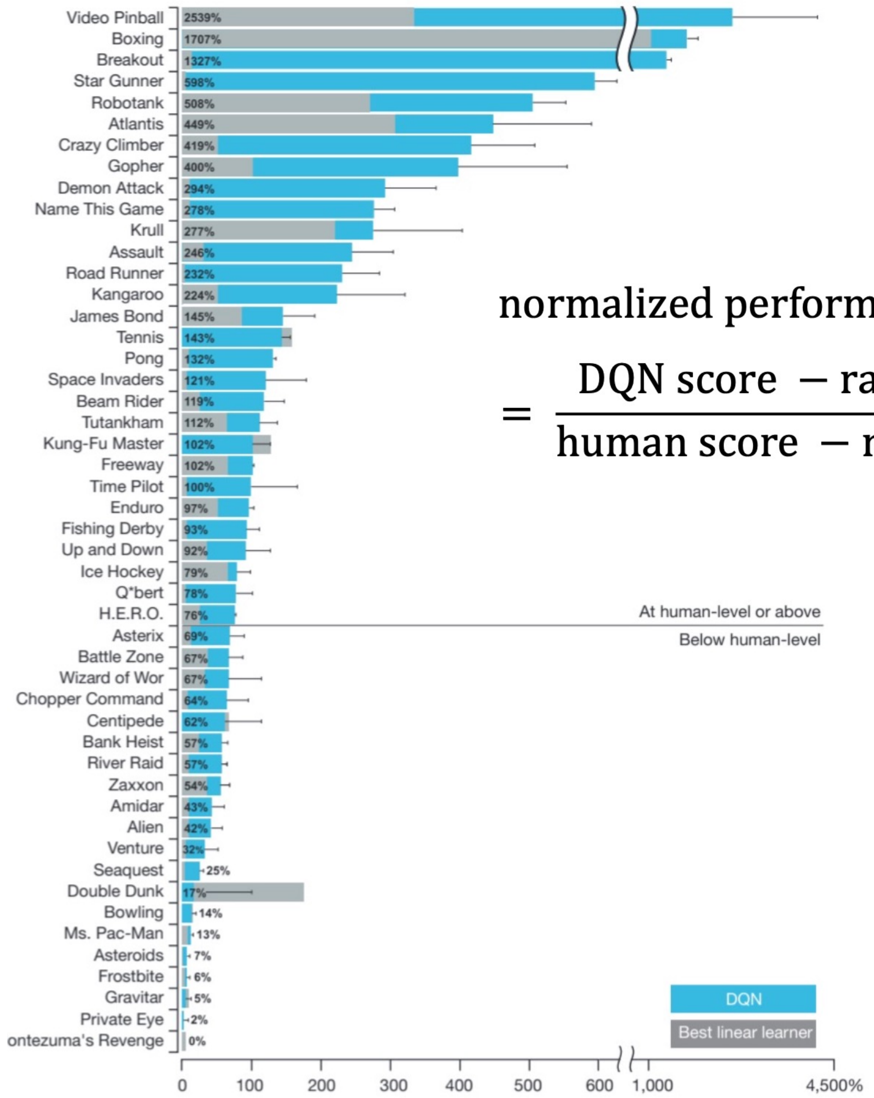
“Human-Level Control Through Deep Reinforcement Learning”, Mnih, Kavukcuoglu, Silver et al. (2015)

DQN training procedure

- Collect transitions using an ε-greedy exploration policy
  - Store $\{(s_t, a_t, s_{t+1}, r_t)\}$ into the replay buffer
- Sample a minibatch of k transitions from the buffer
- Update networks:
  - Compute the target using the sampled transitions
  - Update the evaluation network Q_θ
  - Every C steps, synchronize the target network Q_{θ^-} with the evaluation network

“Human-Level Control Through Deep Reinforcement Learning”, Mnih, Kavukcuoglu, Silver et al. (2015)

DQN performance in Atari games



normalized performance

$$= \frac{\text{DQN score} - \text{random play score}}{\text{human score} - \text{random play score}}$$

The performance of DQN is normalized with respect to a professional human games tester (that is, 100% level)

“Human-Level Control Through Deep Reinforcement Learning”, Mnih, Kavukcuoglu, Silver et al. (2015)

Overestimation in Q-Learning

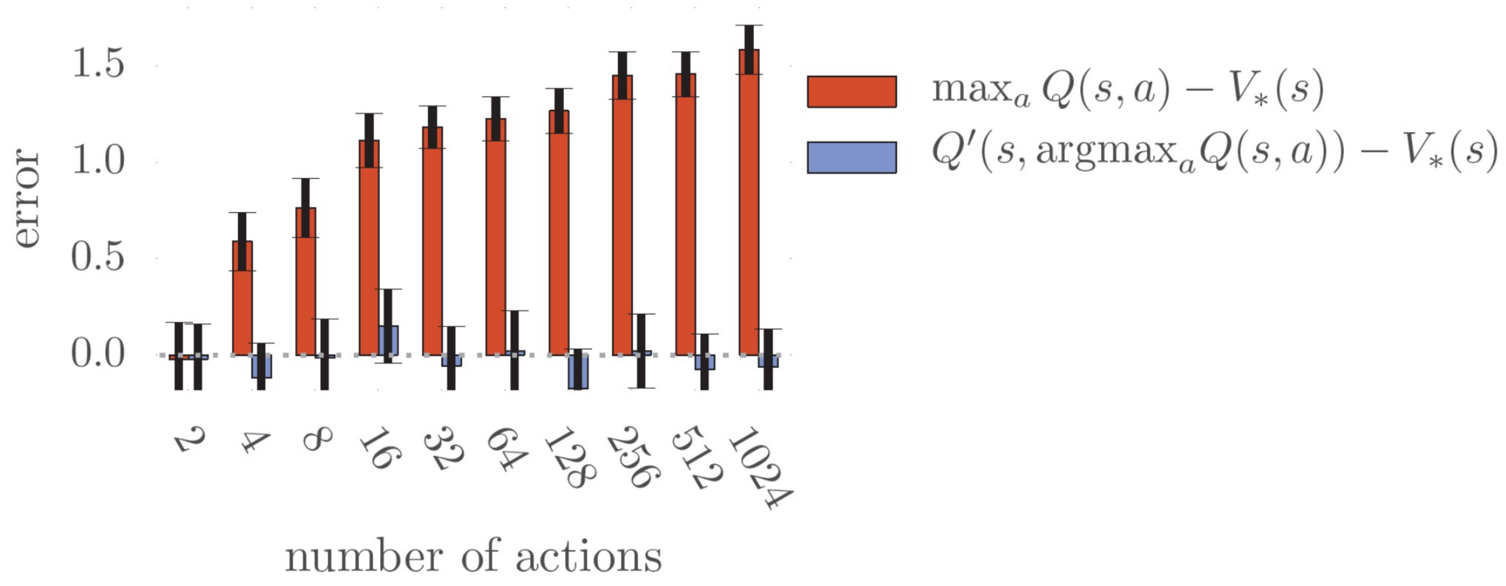
- Q-function overestimation
  - The target value is computed as: $y_t = r_t + \gamma \max_{a'} Q_\theta(s_{t+1}, a')$
  - The max operator leads to increasingly larger Q-values, potentially exceeding the true value
- Cause of overestimation

$$\max_{a' \in A} Q_{\theta'}(s_{t+1}, a') = Q_{\theta'}(s_{t+1}, \arg\max_{a'} Q_{\theta'}(s_{t+1}, a'))$$

  - The chosen action might be overestimated due to Q-function error

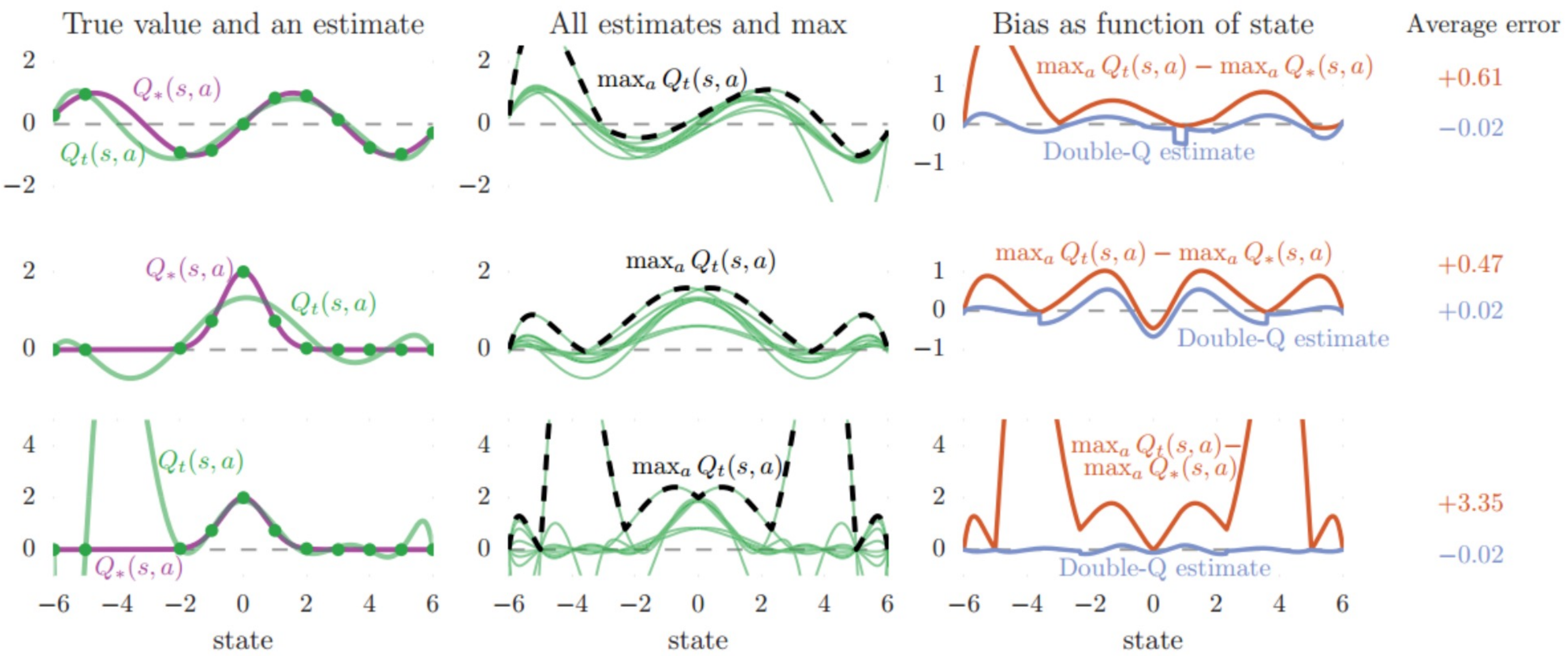
Degree of overestimation in DQN

- Overestimation increases with the number of candidate actions



  - A separately trained Q'-function is used as a reference

Overestimation example in DQN



- Setup: The x-axis represents states, and each plot includes 10 candidate actions. The purple curve denotes the true Q-value function, the green dots are training data points, and the green lines are the fitted Q-value estimates.
- The middle column shows the estimated values $Q_t(s,a)$ for all 10 actions. After applying the max operator, the results deviate significantly from the true values $Q_*(s,a)$.

Double DQN

- Uses two separate networks for action selection and value estimation, respectively.

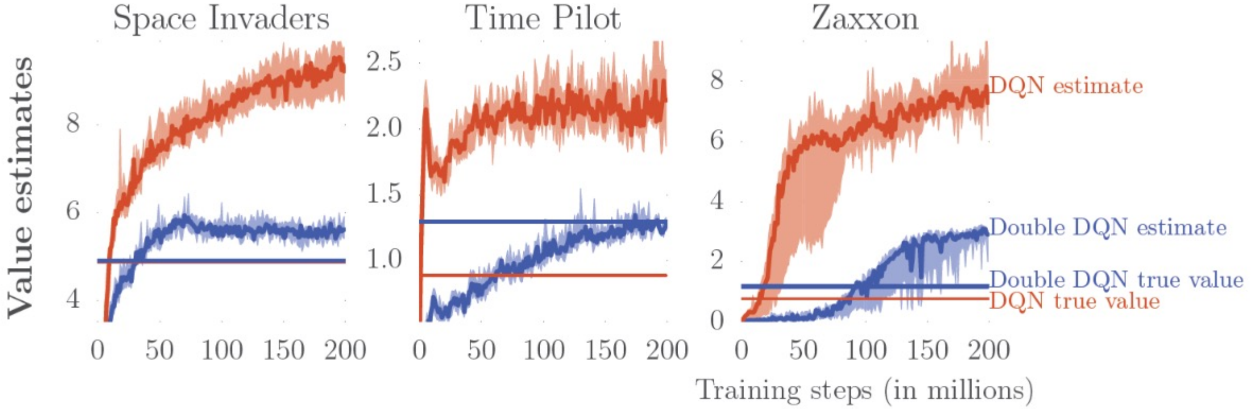
DQN $y_t = r_t + \gamma Q_\theta(s_{t+1}, \arg\max_{a'} Q_\theta(s_{t+1}, a'))$

Double DQN $y_t = r_t + \gamma Q_{\theta'}(s_{t+1}, \arg\max_{a'} Q_\theta(s_{t+1}, a'))$

"Double Reinforcement Learning with Double Q-Learning", van Hasselt et al. (2016)

Experimental results in the Atari environment

- Value estimation error

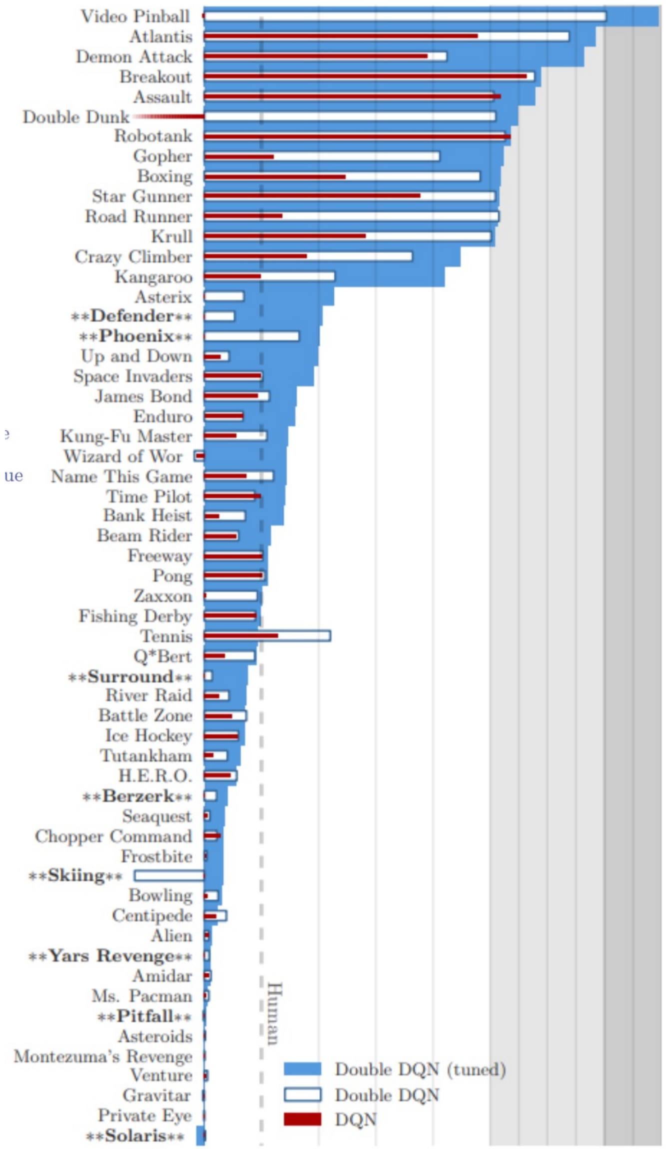


- Atari game performance

	no ops		human starts		
	DQN	DDQN	DQN	DDQN	DDQN (tuned)
Median	93%	**115%**	47%	88%	**117%**
Mean	241%	**330%**	122%	273%	**475%**

normalized performance

$$= \frac{\text{DQN score} - \text{random play score}}{\text{human score} - \text{random play score}}$$



Dueling DQN

- Assume the action-value function follows a distribution:

$$Q(s,a) \sim \mathcal{N}(\mu, \sigma)$$

- Then: $V(s) = \mathbb{E}[Q(s,a)] = \mu$ $\qquad Q(s,a) = \mu + \boxed{\varepsilon(s,a)}$

- How do we describe $\varepsilon(s,a)$?

$$\varepsilon(s,a) = Q(s,a) - V(s)$$

  - This term is also known as the Advantage function

"Dueling Network Architectures for Deep Reinforcement Learning", Wang et al. (2016)

Dueling DQN (cont.)

- Advantage function

$$A^{\pi}(s,a) = Q^{\pi}(s,a) - V^{\pi}(s)$$

$$Q^{\pi}(s,a) = \mathbb{E}[R_t | s_t = s, a_t = a, \pi]$$

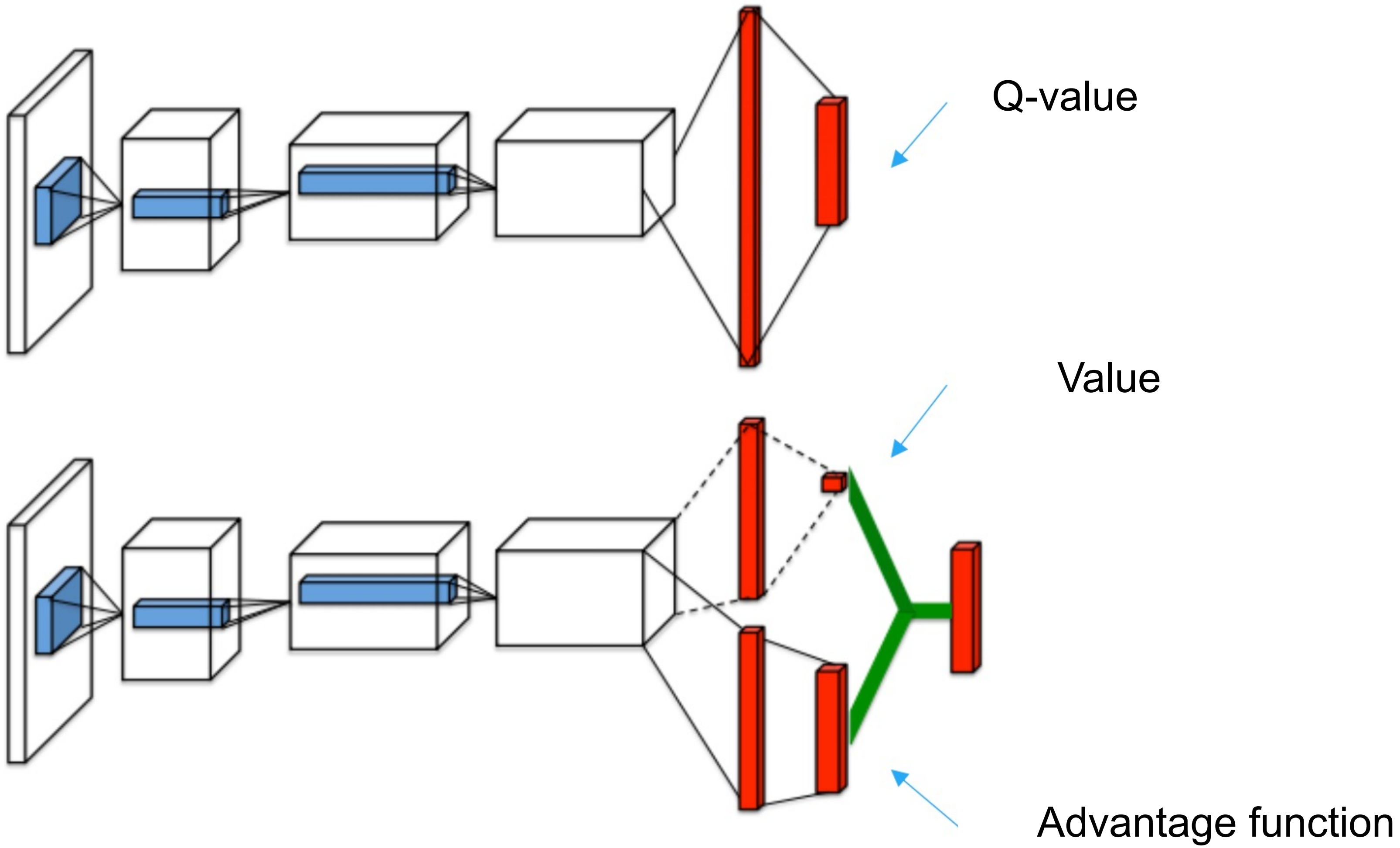
$$V^{\pi}(s) = \mathbb{E}_{a \sim \pi(s)}[Q^{\pi}(s,a)]$$

- Different forms of advantage aggregation

$$Q(s,a;\theta,\alpha,\beta) = V(s;\theta,\beta) + \left(A(s,a;\theta,\alpha) - \max_{a' \in |A|} A(s,a';\theta,\alpha)\right)$$

$$Q(s,a;\theta,\alpha,\beta) = V(s;\theta,\beta) + \left(A(s,a;\theta,\alpha) - \frac{1}{|A|}\sum_{a'} A(s,a';\theta,\alpha)\right)$$

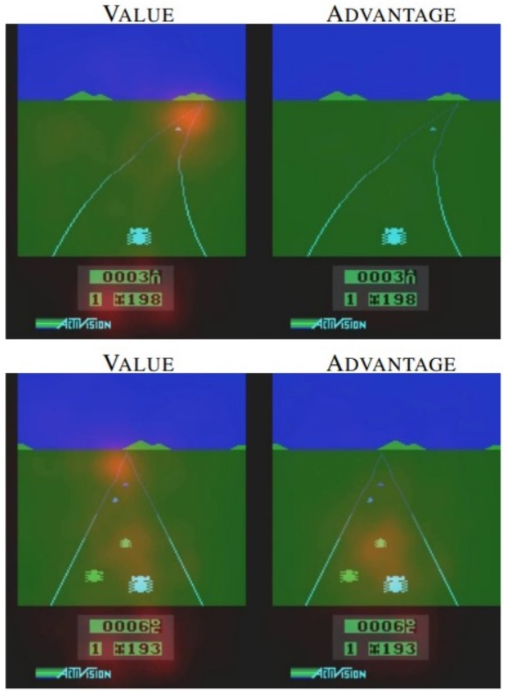
Network structure

- 

Advantages of Dueling DQN

- Effective for states weakly correlated with actions
- More efficient learning of the state-value function
  - The value stream V(s) is shared across all actions, allowing the network to generalize better across actions

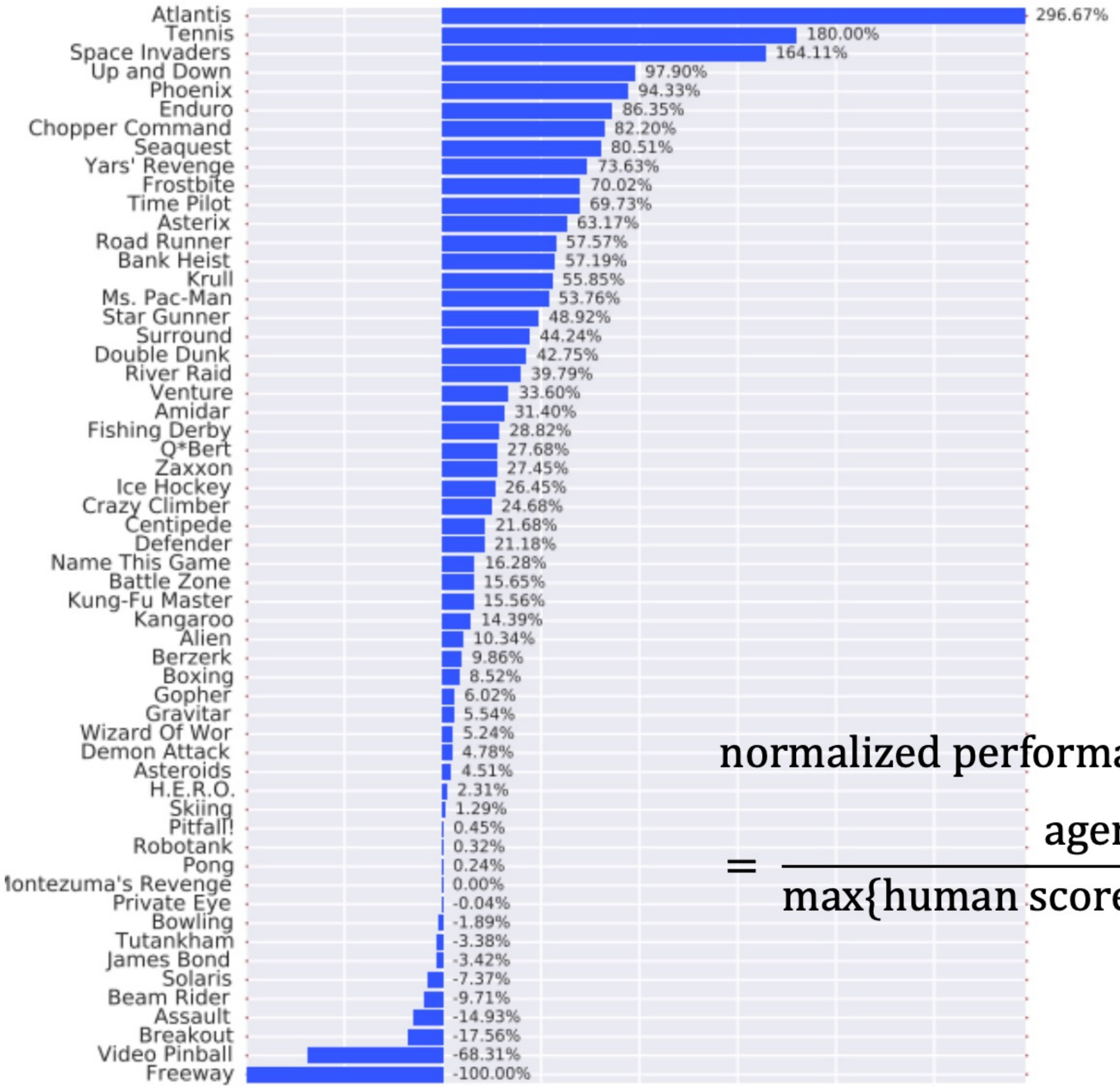
saliency maps



•The value stream allows the agent to evaluate how good a state is without considering the specific action taken.

•The advantage stream emphasizes action-specific importance: for instance, it can learn to focus more when an obstacle (e.g., a car) appears in front of the agent, thereby guiding more precise action selection.

Experimental results in the Atari environment I

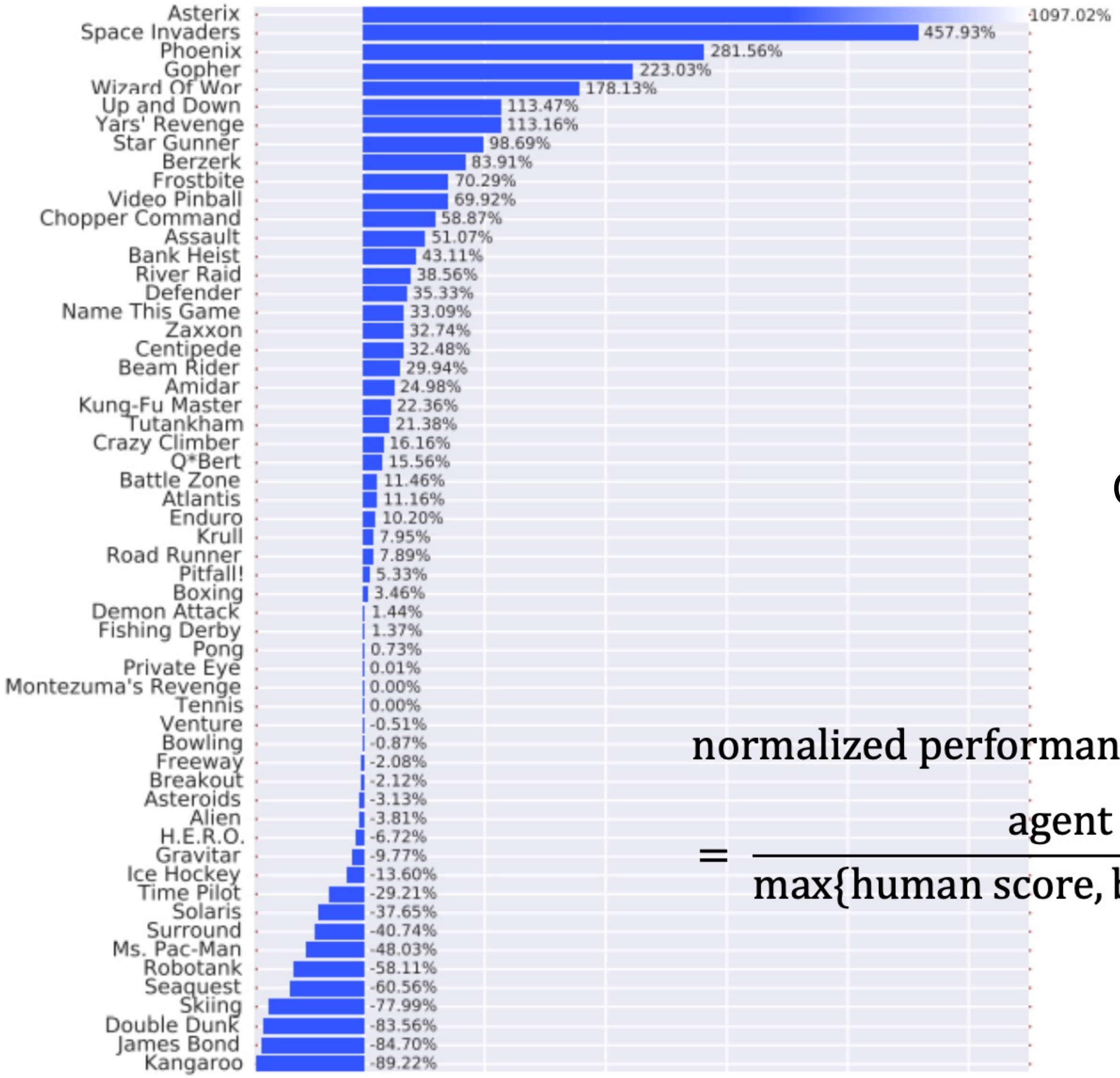


Compared with DQN

normalized performance

$$= \frac{\text{agent score} - \text{baseline score}}{\max\{\text{human score}, \text{baseline score}\} - \text{random play score}}$$

Experimental results in the Atari environment II



Compared with Double DQN

normalized performance

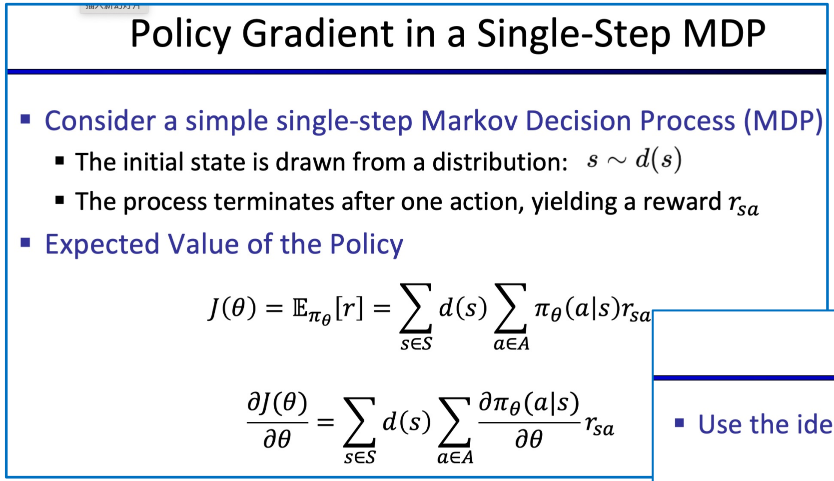
$$= \frac{\text{agent score} - \text{baseline score}}{\max\{\text{human score}, \text{baseline score}\} - \text{random play score}}$$

Deep RL – Policy-based methods

Review: The policy gradient theorem

- The policy gradient theorem generalizes the derivation of likelihood ratios to the multi-step MDP setting.
- It replaces the immediate reward r_t with the expected long-term return $Q^{\pi}(s, a)$.

$$\frac{\partial J(\theta)}{\partial \theta} = \mathbb{E}_{\pi_\theta}\left[\frac{\partial \log \pi_\theta(a|s)}{\partial \theta} Q^{\pi_\theta}(s,a)\right]$$

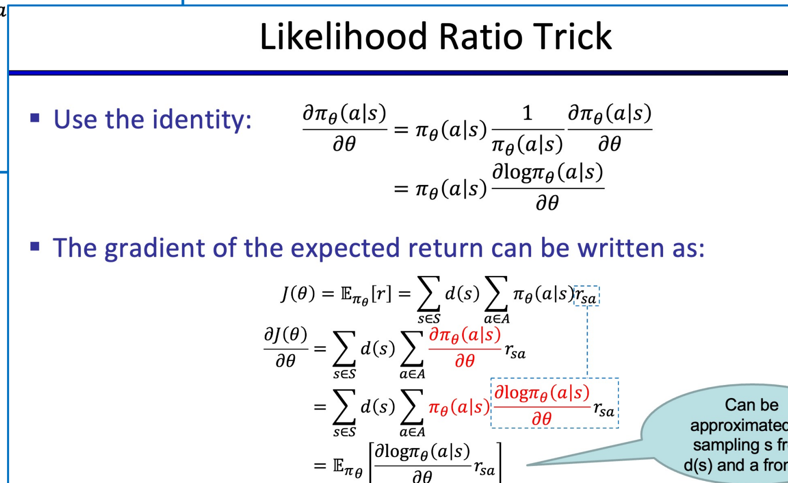


Policy Gradient in a Single-Step MDP

- Consider a simple single-step Markov Decision Process (MDP)
  - The initial state is drawn from a distribution: $s \sim d(s)$
  - The process terminates after one action, yielding a reward r_{sa}
- Expected Value of the Policy

$$J(\theta) = \mathbb{E}_{\pi_\theta}[r] = \sum_{s \in S} d(s) \sum_{a \in A} \pi_\theta(a|s) r_{sa}$$

$$\frac{\partial J(\theta)}{\partial \theta} = \sum_{s \in S} d(s) \sum_{a \in A} \frac{\partial \pi_\theta(a|s)}{\partial \theta} r_{sa}$$



Likelihood Ratio Trick

- Use the identity:

$$\frac{\partial \pi_\theta(a|s)}{\partial \theta} = \pi_\theta(a|s) \frac{1}{\pi_\theta(a|s)} \frac{\partial \pi_\theta(a|s)}{\partial \theta} = \pi_\theta(a|s) \frac{\partial \log \pi_\theta(a|s)}{\partial \theta}$$

- The gradient of the expected return can be written as:

$$J(\theta) = \mathbb{E}_{\pi_\theta}[r] = \sum_{s \in S} d(s) \sum_{a \in A} \pi_\theta(a|s) r_{sa}$$

$$\frac{\partial J(\theta)}{\partial \theta} = \sum_{s \in S} d(s) \sum_{a \in A} \frac{\partial \pi_\theta(a|s)}{\partial \theta} r_{sa} = \sum_{s \in S} d(s) \sum_{a \in A} \pi_\theta(a|s) \frac{\partial \log \pi_\theta(a|s)}{\partial \theta} r_{sa} = \mathbb{E}_{\pi_\theta}\left[\frac{\partial \log \pi_\theta(a|s)}{\partial \theta} r_{sa}\right]$$

Review: Policy Gradient in a Single-Step MDP

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$$= \sum_{s \in S} d(s) \sum_{a \in A} \pi_\theta(a|s) \frac{\partial \log \pi_\theta(a|s)}{\partial \theta} r_{sa}$$

$$= \mathbb{E}_{\pi_\theta}\left[\frac{\partial \log \pi_\theta(a|s)}{\partial \theta} r_{sa}\right]$$

Can be approximated by sampling s from d(s) and a from π_θ

Policy network gradient

- For stochastic policies, the probability of selecting an action is typically modeled using a softmax function:

$$\pi_\theta(a|s) = \frac{e^{f_\theta(s,a)}}{\sum_{a'} e^{f_\theta(s,a')}}$$

  - $f_\theta(s,a)$ is a score function (e.g., logits) for the state-action pair
  - Parameterized by θ, often realized via a neural network
- Gradient of the log-form

$$\frac{\partial \log \pi_\theta(a|s)}{\partial\theta} = \frac{\partial f_\theta(s,a)}{\partial\theta} - \frac{1}{\sum_{a'} e^{f_\theta(s,a')}} \sum_{a''} e^{f_\theta(s,a'')} \frac{\partial f_\theta(s,a'')}{\partial\theta}$$

$$= \frac{\partial f_\theta(s,a)}{\partial\theta} - \mathbb{E}_{a'\sim\pi_\theta(a'|s)}\left[\frac{\partial f_\theta(s,a')}{\partial\theta}\right]$$

Policy network gradient (cont.)

- Gradient of the log-form

$$\frac{\partial \log \pi_\theta(a|s)}{\partial \theta} = \frac{\partial f_\theta(s,a)}{\partial \theta} - \mathbb{E}_{a' \sim \pi_\theta(a'|s)}\left[\frac{\partial f_\theta(s,a')}{\partial \theta}\right]$$

- Gradient of the policy network

$$\frac{\partial J(\theta)}{\partial \theta} = \mathbb{E}_{\pi_\theta}\left[\frac{\partial \log \pi_\theta(a|s)}{\partial \theta} Q^{\pi_\theta}(s,a)\right]$$

$$= \mathbb{E}_{\pi_\theta}\left[\left(\frac{\partial f_\theta(s,a)}{\partial \theta} - \mathbb{E}_{a' \sim \pi_\theta(a'|s)}\left[\frac{\partial f_\theta(s,a')}{\partial \theta}\right]\right) Q^{\pi_\theta}(s,a)\right]$$

1. Rollout
2. Train another network

Back propagation

Back propagation

Comparison: DQN v.s. Policy gradient

- Q-Learning:
  - Learns a Q-value function $Q_\theta(s, a)$ parameterized by θ
  - Objective: Minimize the TD error

$$J(\theta) = \mathbb{E}_{\pi'}\left[\frac{1}{2}\left(r_t + \gamma \max_{a'} Q_{\theta'}(s_{t+1}, a') - Q_\theta(s_t, a_t)\right)^2\right]$$

$$\theta \leftarrow \theta - \alpha \frac{\partial J(\theta)}{\partial \theta}$$

$$= \theta + \alpha \mathbb{E}_{\pi'}\left[\left(r_t + \gamma \max_{a'} Q_{\theta'}(s_{t+1}, a') - Q_\theta(s_t, a_t)\right) \frac{\partial Q_\theta(s, a)}{\partial \theta}\right]$$

Comparison: DQN v.s. Policy gradient

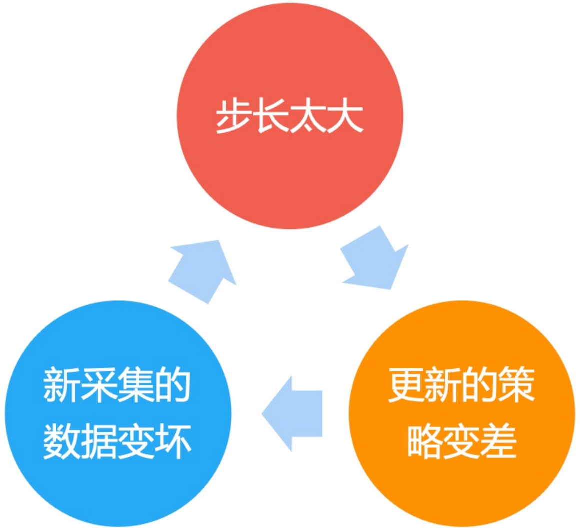
- Q-Learning:
  - Learns a Q-value function $Q_\theta(s, a)$ parameterized by θ
  - Objective: Minimize the TD error
- Policy gradient
  - Learns a policy $\pi_\theta(a \mid s)$ directly, parameterized by θ
  - Objective: Maximize the expected return directly

$$\max_\theta J(\theta) = \mathbb{E}_{\pi_\theta}[Q^{\pi_\theta}(s, a)]$$

$$\theta \leftarrow \theta + \alpha \frac{\partial J(\theta)}{\partial \theta} = \theta + \alpha \mathbb{E}_{\pi_\theta}\left[\frac{\partial \log \pi_\theta(a|s)}{\partial \theta} Q^{\pi_\theta}(s, a)\right]$$

Limitations of policy gradient methods

- Learning rate (step size) selection is challenging in policy gradient algorithms
  - Since the data distribution changes as the policy updates, a previously good learning rate may become ineffective.
  - A poor choice of step size can significantly degrade performance:
    - Too large → policy diverges or collapses
    - Too small → slow convergence or stagnation



Trust Region Policy Optimization (TRPO)

- Two forms of the optimization objective
  - Form 1: Trajectory-based objective $J(\theta) = \mathbb{E}_{\tau \sim p_\theta(\tau)}[\sum_t \gamma^t r(s_t, a_t)]$
  - Form 2: State-value-based objective $J(\theta) = \mathbb{E}_{s_0 \sim p_\theta(s_0)}[V^{\pi_\theta}(s_0)]$

Optimization gap of the objective function

$$J(\theta) = \mathbb{E}_{\tau \sim p_\theta(\tau)}[\sum_t \gamma^t r(s_t, a_t)]$$
$$J(\theta) = \mathbb{E}_{s_0 \sim p_\theta(s_0)}[V^{\pi_\theta}(s_0)]$$

- New policy θ' and old policy θ

$$J(\theta') - J(\theta) = J(\theta') - \mathbb{E}_{s_0 \sim p(s_0)}[V^{\pi_\theta}(s_0)]$$

$$= \mathbb{E}_{\tau \sim p_{\theta'}(\tau)}[\sum_{t=0} \gamma^t A^{\pi_\theta}(s_t, a_t)]$$

Sampling inconvenience

$$A^{\pi_\theta}(s_t, a_t) = Q^{\pi_\theta}(s_t, a_t) - V^{\pi_\theta}(s_t)$$

Review: Policy Iteration (PI) in MDP

- Evaluation: For fixed current policy π, find values with policy evaluation:
  - Iterate until values converge:

$$V_{k+1}^{\pi_i}(s) \leftarrow \sum_{s'} T(s, \pi_i(s), s') \left[R(s, \pi_i(s), s') + \gamma V_k^{\pi_i}(s')\right]$$

- Improvement: For fixed values, get a better policy using policy extraction
  - One-step look-ahead:

$$\pi_{i+1}(s) = \arg\max_a \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma V^{\pi_i}(s')\right]$$

1. Improvement: Does each policy improvement step produce a better policy?
2. Convergence: Does PI converge to an optimal policy?

Importance sampling

$$A^{\pi_\theta}(s_t, a_t) = Q^{\pi_\theta}(s_t, a_t) - V^{\pi_\theta}(s_t)$$

$$J(\theta') - J(\theta)$$

$$= \mathbb{E}_{\tau \sim p_{\theta'}(\tau)}\left[\sum_{t=0}^{\infty} \gamma^t A^{\pi_\theta}(s_t, a_t)\right]$$

$$= \sum_t \mathbb{E}_{s_t \sim p_{\theta'}(s_t)}[\mathbb{E}_{a_t \sim \pi_{\theta'}(a_t|s_t)}[\gamma^t A^{\pi_\theta}(s_t, a_t)]]$$

$$= \sum_t \mathbb{E}_{s_t \sim p_{\theta'}(s_t)}[\mathbb{E}_{a_t \sim \pi_\theta(a_t|s_t)}[\frac{\pi_{\theta'}(a_t|s_t)}{\pi_\theta(a_t|s_t)} \gamma^t A^{\pi_\theta}(s_t, a_t)]]$$

$p_{\theta'}$, approximation

Importance sampling

Ignoring the difference in state distributions

- When the change between the old policy π_θ and the new policy $\pi_{\theta'}$ is small, we can approximate $p_\theta(s_t) \approx p_{\theta'}(s_t)$
  - For deterministic policies:
    The probability that $\pi_{\theta'}(s_t) \neq \pi_\theta\ (s_t)$ is less than a small threshold ϵ.
  - For stochastic policies:
    The probability that $a' \sim \pi_{\theta'}(\cdot\ |s_t) \neq a \sim \pi_\theta(\cdot\ |s_t)$ is less than ϵ.

$$J(\theta') - J(\theta) \approx \sum_t \mathbb{E}_{s_t \sim p_\theta(s_t)}[\mathbb{E}_{a_t \sim \pi_\theta(a_t|s_t)}[\frac{\pi_{\theta'}(a_t|s_t)}{\pi_\theta(a_t|s_t)} \gamma^t A^{\pi_\theta}(s_t, a_t)]]$$

TRPO Policy Constraint

- Use KL divergence to constrain policy update magnitude:

$$\theta' \leftarrow \arg\max_{\theta'} \sum_t \mathbb{E}_{s_t \sim p_\theta(s_t)}[\mathbb{E}_{a_t \sim \pi_\theta(a_t|s_t)}[\frac{\pi_{\theta'}(a_t|s_t)}{\pi_\theta(a_t|s_t)} \gamma^t A^{\pi_\theta}(s_t, a_t)]]$$

$$\text{such that } \mathbb{E}_{s_t \sim p_\theta(s_t)}[D_{KL}(\pi_{\theta'}(a_t|s_t) \parallel \pi_\theta(a_t|s_t))] \leq \epsilon$$

  - In practice: use penalized objective with KL divergence penalty instead of hard constraint

$$\theta' \leftarrow \arg\max_{\theta'} \sum_t \mathbb{E}_{s_t \sim p_\theta(s_t)}[\mathbb{E}_{a_t \sim \pi_\theta(a_t|s_t)}[\frac{\pi_{\theta'}(a_t|s_t)}{\pi_\theta(a_t|s_t)} \gamma^t A^{\pi_\theta}(s_t, a_t)]] - \lambda(D_{KL}(\pi_{\theta'}(a_t|s_t) \parallel \pi_\theta(a_t|s_t)) - \epsilon)$$

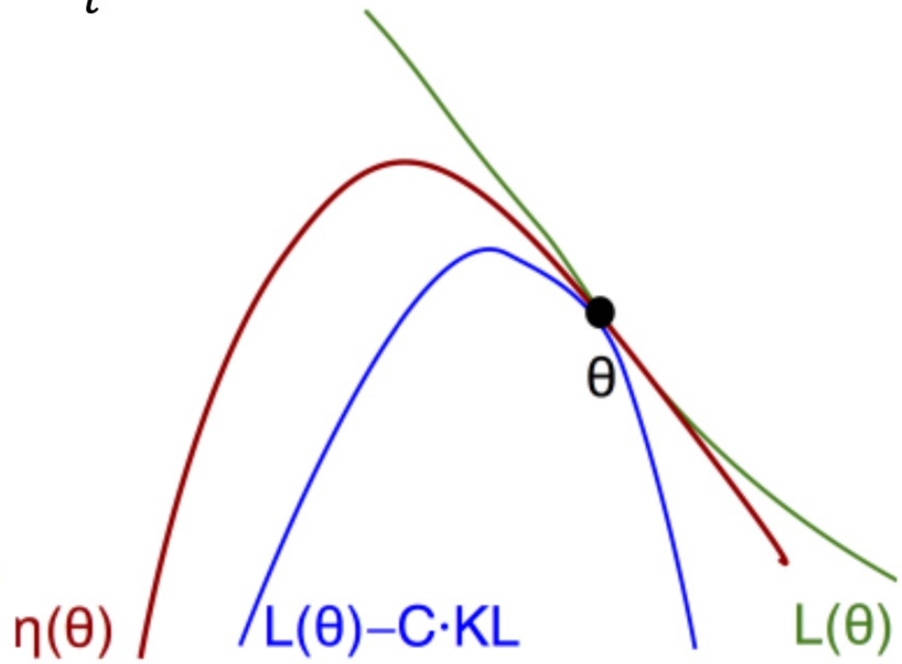
    - Update θ' and $\lambda \leftarrow \lambda + \alpha(D_{KL}(\pi_{\theta'}(a_t|s_t) \parallel \pi_\theta(a_t|s_t)) - \epsilon)$

TRPO Monotonic Improvement Guarantee

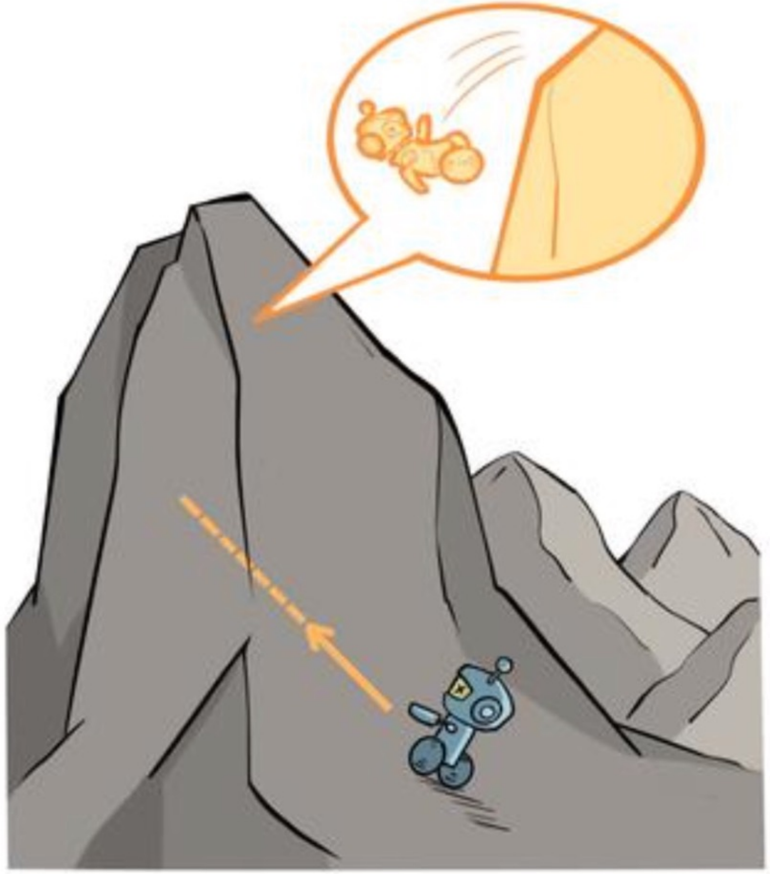
$$J(\theta') \geq L_\theta(\theta') - C \cdot D_{KL}^{max}(\theta, \theta'), where\ C = \frac{4\epsilon\gamma}{(1-\gamma)^2}, \epsilon = \max_{s,a} |A_\pi(s,a)|$$

$$L_\theta(\theta') = J(\theta) + \sum_t \mathbb{E}_{s_t \sim p_\theta(s_t)}[\mathbb{E}_{a_t \sim \pi_\theta(a_t|s_t)}[\frac{\pi_{\theta'}(a_t|s_t)}{\pi_\theta(a_t|s_t)}\gamma^t A^{\pi_\theta}(s_t, a_t)]]$$

$J(\theta')$



Principle of TRPO



Gradient Ascent

Optimization in Trust Region

TRPO Drawbacks

Use KL divergence to constrain policy update magnitude:

$$\theta' \leftarrow \arg\max_{\theta'} \sum_t \mathbb{E}_{s_t \sim p_\theta(s_t)}[\mathbb{E}_{a_t \sim \pi_\theta(a_t|s_t)}[\frac{\pi_{\theta'}(a_t|s_t)}{\pi_\theta(a_t|s_t)} \gamma^t A^{\pi_\theta}(s_t, a_t)]]$$

$$\text{such that } \mathbb{E}_{s_t \sim p_\theta(s_t)}[D_{KL}\left(\pi_{\theta'}(a_t|s_t) \parallel \pi_\theta(a_t|s_t)\right)] \leq \epsilon$$

- In practice: use penalized objective with KL divergence penalty instead of hard constraint

$$\theta' \leftarrow \arg\max_{\theta'} \sum_t \mathbb{E}_{s_t \sim p_\theta(s_t)}[\mathbb{E}_{a_t \sim \pi_\theta(a_t|s_t)}[\frac{\pi_{\theta'}(a_t|s_t)}{\pi_\theta(a_t|s_t)} \gamma^t A^{\pi_\theta}(s_t, a_t)]] - \lambda(D_{KL}(\pi_{\theta'}(a_t|s_t) \parallel \pi_\theta(a_t|s_t)) - \epsilon)$$

  - Update θ' and $\lambda \leftarrow \lambda + \alpha(D_{KL}(\pi_{\theta'}(a_t|s_t) \parallel \pi_\theta(a_t|s_t)) - \epsilon)$

- High variance from importance weights
- Difficult to solve constrained optimization

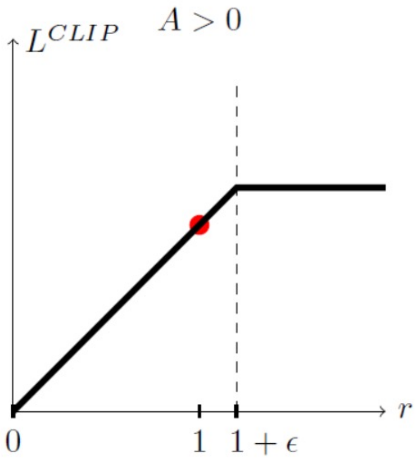
Proximal Policy Optimization (PPO)

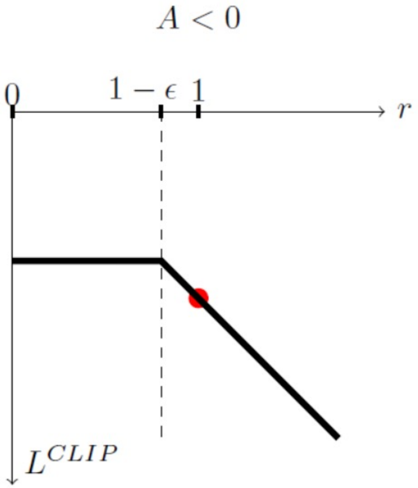
- Clipped Surrogate Objective

conservative policy iteration

$$L^{CPI}(\theta) = \hat{\mathbb{E}}_t\left[\frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_{\text{old}}}(a_t|s_t)}\hat{A}_t\right] = \hat{\mathbb{E}}_t\left[r_t(\theta)\hat{A}_t\right]$$

$$L^{CLIP}(\theta) = \hat{\mathbb{E}}_t\left[\min\left(r_t(\theta)\hat{A}_t, \text{clip}(r_t(\theta), 1-\epsilon, 1+\epsilon)\hat{A}_t\right)\right]$$





Construct the lower bound: $L^{CLIP}(\theta) \leq L^{CPI}(\theta)$

Equivalent at r=1: $L^{CLIP}(\theta) = L^{CPI}(\theta)$

PPO: improvement over TRPO

- 1.Clipped surrogate objective

conservative policy iteration

$$L^{CPI}(\theta) = \hat{\mathbb{E}}_t \left[\frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_{\text{old}}}(a_t|s_t)} \hat{A}_t \right] = \hat{\mathbb{E}}_t \left[r_t(\theta) \hat{A}_t \right]$$

$$L^{CLIP}(\theta) = \hat{\mathbb{E}}_t \left[\min\left(r_t(\theta)\hat{A}_t, \text{clip}(r_t(\theta), 1-\epsilon, 1+\epsilon)\hat{A}_t \right) \right]$$

PPO: improvement over TRPO

- 2. Adaptive penalty parameter

$$L^{KLPEN}(\theta) = \hat{\mathbb{E}}_t\left[\frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_{\text{old}}}(a_t|s_t)}\hat{A}_t - \beta \mathrm{KL}[\pi_{\theta_{\text{old}}}(\cdot\,|s_t)|\pi_\theta(\cdot\,|s_t)]\right]$$

  - Adjust the penalty coefficient β dynamically:
    - Compute the KL value $d = \hat{\mathbb{E}}_t\left[\mathrm{KL}\left[\pi_{\theta_{\text{old}}}(\cdot\,|s_t)\middle|\pi_\theta(\cdot\,|s_t)\right]\right]$
    - If d < target / 1.5 → β← β / 2
    - If d > target × 1.5 → β← β × 2

Note: Here, 1.5 and 2 are empirical parameters, and the algorithm performance is not very sensitive to them

PPO experimental comparison

No clipping or penalty: $L_t(\theta) = r_t(\theta)\hat{A}_t$

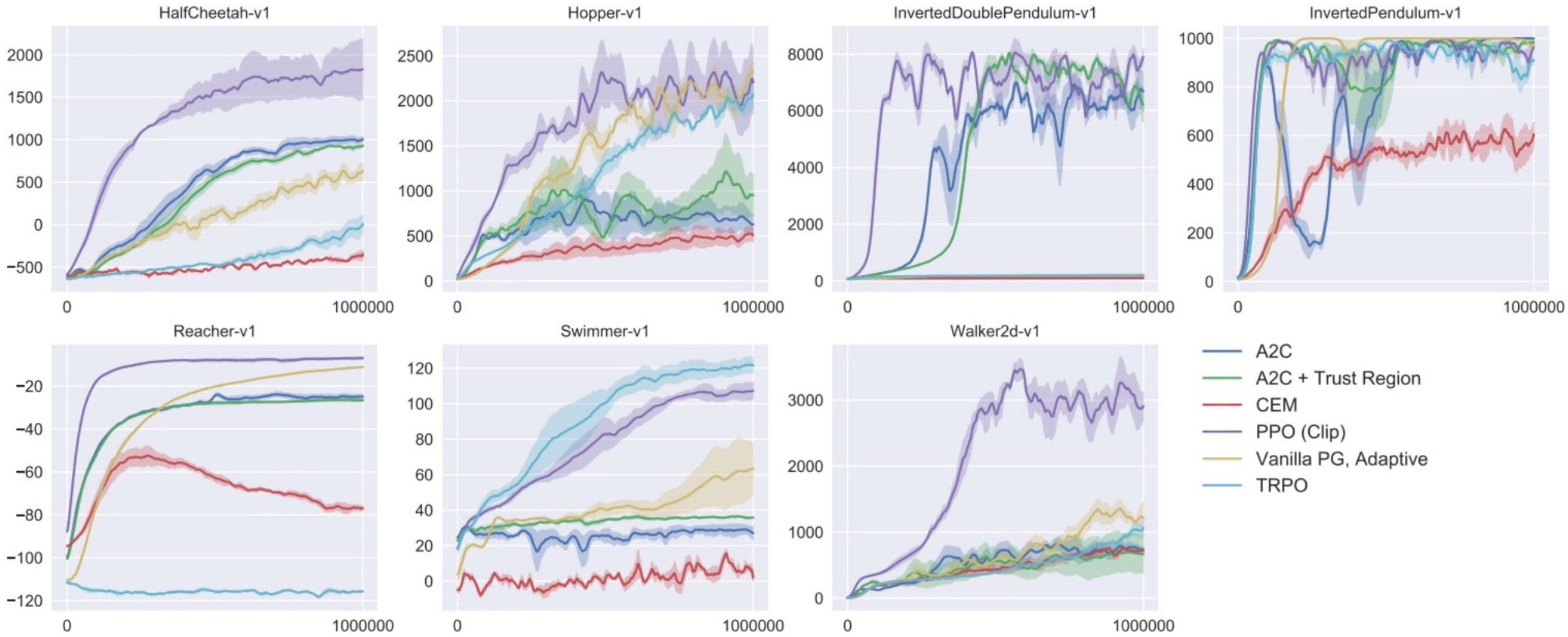
Clipping: $L_t(\theta) = \min(r_t(\theta)\hat{A}_t, \text{clip}(r_t(\theta)), 1-\epsilon, 1+\epsilon)\hat{A}_t$

KL penalty (fixed or adaptive) $L_t(\theta) = r_t(\theta)\hat{A}_t - \beta\,\text{KL}[\pi_{\theta_{\text{old}}}, \pi_\theta]$

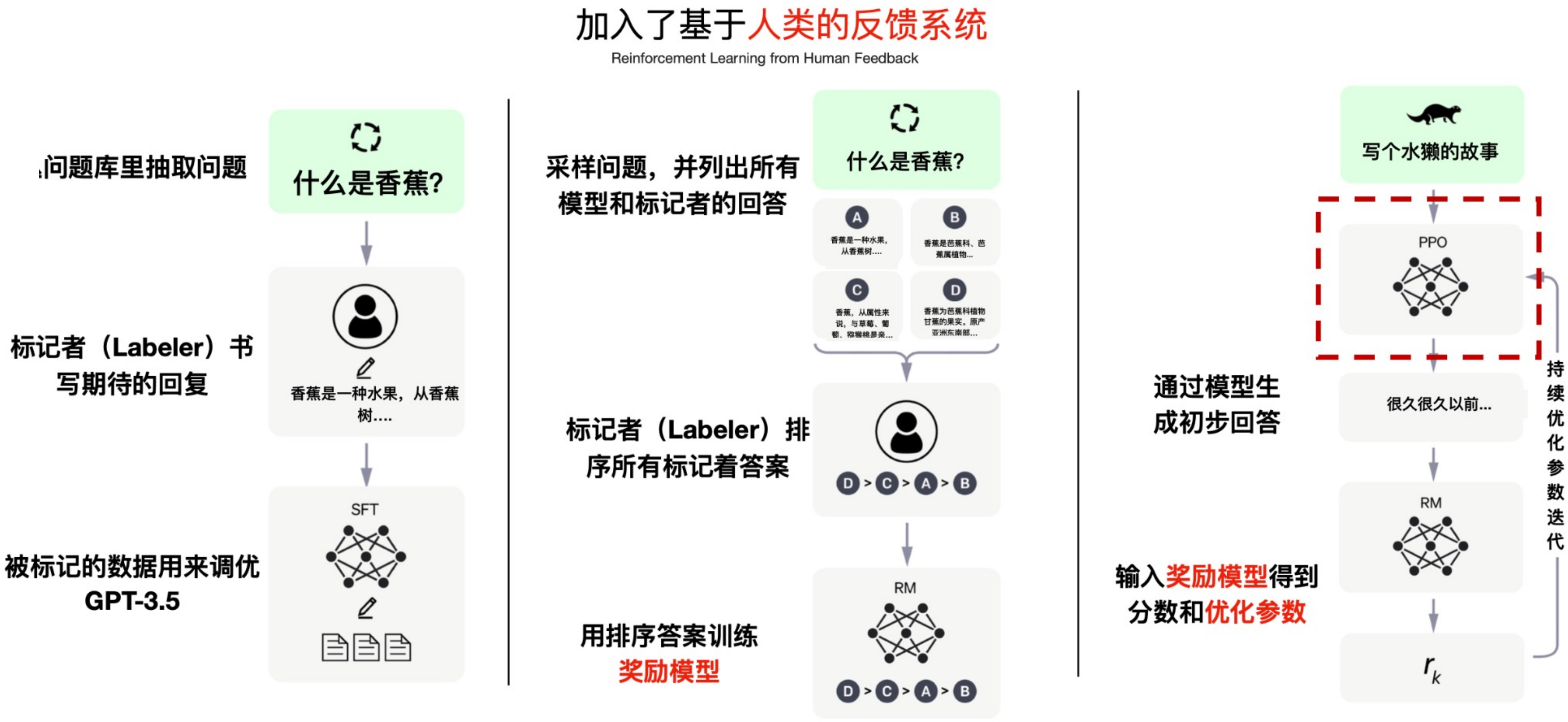
7 continuous control environments
3 random seeds
Each algorithm runs 100 episodes, repeated 21 times
Scores normalized to best model achieving 1.0

algorithm	avg. normalized score
No clipping or penalty	-0.39
Clipping, $\epsilon = 0.1$	0.76
Clipping, $\epsilon = 0.2$	**0.82**
Clipping, $\epsilon = 0.3$	0.70
Adaptive KL $d_{\text{targ}} = 0.003$	0.68
Adaptive KL $d_{\text{targ}} = 0.01$	0.74
Adaptive KL $d_{\text{targ}} = 0.03$	0.71
Fixed KL, $\beta = 0.3$	0.62
Fixed KL, $\beta = 1.$	0.71
Fixed KL, $\beta = 3.$	0.72
Fixed KL, $\beta = 10.$	0.69

PPO in MuJoCo



PPO in ChatGPT



GRPO in deepseek

Group Relative Policy Optimization In order to save the training costs of RL, we adopt Group Relative Policy Optimization (GRPO) (Shao et al., 2024), which foregoes the critic model that is typically the same size as the policy model, and estimates the baseline from group scores instead. Specifically, for each question q, GRPO samples a group of outputs $\{o_1, o_2, \cdots, o_G\}$ from the old policy $\pi_{\theta_{old}}$ and then optimizes the policy model π_θ by maximizing the following objective:

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E}[q \sim P(Q), \{o_i\}_{i=1}^G \sim \pi_{\theta_{old}}(O|q)]$$
$$\frac{1}{G}\sum_{i=1}^{G}\left(\min\left(\frac{\pi_\theta(o_i|q)}{\pi_{\theta_{old}}(o_i|q)}A_i, \text{clip}\left(\frac{\pi_\theta(o_i|q)}{\pi_{\theta_{old}}(o_i|q)}, 1-\varepsilon, 1+\varepsilon\right)A_i\right) - \beta\mathbb{D}_{KL}\left(\pi_\theta||\pi_{ref}\right)\right), \quad (1)$$

$$\mathbb{D}_{KL}\left(\pi_\theta||\pi_{ref}\right) = \frac{\pi_{ref}(o_i|q)}{\pi_\theta(o_i|q)} - \log\frac{\pi_{ref}(o_i|q)}{\pi_\theta(o_i|q)} - 1, \quad (2)$$

where ε and β are hyper-parameters, and A_i is the advantage, computed using a group of rewards $\{r_1, r_2, \ldots, r_G\}$ corresponding to the outputs within each group:

$$A_i = \frac{r_i - mean(\{r_1, r_2, \cdots, r_G\})}{std(\{r_1, r_2, \cdots, r_G\})}. \quad (3)$$

Summary

- 1. Value-based deep RL
  - DQN
  - Double DQN
  - Dueling DQN
- 2. Policy-based RL
  - Policy gradient
  - TRPO
  - PPO
  - GRPO